11. PROFIT AND LOSS

IMPORTANT FACTS

COST PRICE: THE PRICE AT WHICH ARTICLE IS PURCHASED. ABBREVIATED AS C.P.

SELLING PRICE: THE PRICE AT WHICH ARTICLE IS SOLD.

PROFIT OR GAIN: IF SP IS GREATER THAN CP, THE SELLING PRICE IS SAID TO HAVE PROFIT OR GAIN.

LOSS: IF SP IS LESS THAN CP, THE SELLER IS SAID TO INCURED A LOSS.

FORMULA
1. GAIN = (SP) - (CP).
2. LOSS = (CP) - (SP).
3. LOSS OR GAIN IS ALWAYS RECKONED ON CP.
4. GAIN % = (GAIN*100)/CP.
5. LOSS % = (LOSS*100)/CP.
6. SP = ((100+GAIN%) / 100) * CP.
7. SP = ((100-LOSS%) / 100) * CP.
8. CP = (100/(100+GAIN%)) * SP.
9. CP = 100/(100-LOSS%) * SP.
10. IF THE ARTICLE IS SOLD AT A GAIN OF SAY 35%, THEN SP = 135% OF CP.
11. IF A ARTICLE IS SOLD AT A LOSS OF SAY 35%, THEN SP = 65% OF CP.
12. WHEN A PERSON SELLS TWO ITEMS, ONE AT A GAIN OF X% AND OTHER AT A LOSS OF X%. THEN THE SELLER ALWAYS INCURES A LOSS GIVEN: {LOSS% = (COMMON LOSS AND GAIN ^ 2)/10. = (X/10) ^ 2}
13. IF THE TRADER PROFESSES TO SELL HIS GOODS AT CP BUT USES FALSE WEIGHTS, THEN
GAIN=[ERROR/(TRUE VALUE)-(ERROR)*100]%

SOLVED PROBLEMS

ex.1 A man buys an article for rs.27.50 and sells it for rs.28.50. find his gain %.
   sol. cp=rs27.50, sp=rs 28.50
   gain=rs(28.50 -27.50)=rs1.10
   so gain%={(1.10/27.50)*100}=4%

Ex.2. If the a radio is sold for rs 490 and sold for rs 465.50.find loss%.
   sol. cp=rs490, sp= 465.50.
   loss=rs(490-465.50)=rs 24.50.
   loss%=[(24.50/490)*100]%=[(1/2)*100]%=5%

Ex.3.find S.P when
   (i)CP=56.25,gain=20%.
   sol.
   (i)SP =20% of rs 56.25 ,=rs{(120/100)*56.25}=rs67.50.
   (ii)CP=rs 80.40,loss=5%
   sol: sp=85% of rs 80.40
   =rs {(85/100)*80.40}=rs 68.34.

ex.4 find cp when:
   (i) sp =rs 40.60 : gain=16%
   (ii) sp=rs51.70:loss=12%

   (i) cp=rs{(100/116)*40.60}=rs 35.
   (ii) cp=rs{(100/88)*51.87}=rs58.75.

ex.5 A person incures loss for by selling a watch for rs1140.at what price should the
   watch be sold to earn a 5% profit ?
   sol. let the new sp be rsx.then
   (100-loss%): (1st sp)=(100+gain%): (2nd sp)
   ⇐ {(100-5)/1400}={(100+5)/x}=> x={(105*1140)/95} =1260.
   ⇐

ex.6 A book was sold for rs 27.50 with a profit of 10%. if it were sold for rs25.75,
   then what would be % of profit or loss?
   sol. SP=rs 27.50: profit =10%.
   sol. CP=rs {(100/110)*27.50}=rs 25.
   When sp =Rs25.75 ,profit =Rs(25.75-25)=Rs 0.75
   Profit%={(0.75/25)*100}%=25/6%=3%
Ex7. If the cost price is 96% of sp then what is the profit %
   Sol. sp=Rs100 : then cp=Rs 96; profit =Rs 4.
   Profit={4/96}*100}%=4.17%

Ex8. The cp of 21 articles is equal to sp of 18 articles. Find gain or loss %
   CP of each article be Rs 1
   CP of 18 articles =Rs18 , sp of 18 articles =Rs 21.
   Gain%=[(3/18)*100]% =50/3%

Ex9. By selling 33 metres of cloth, one gains the selling price of 11 metres. Find the gain percent.
   Sol:
   (SP of 33m)-(CP of 33m)=Gain=SP of 11m
   SP of 22m = CP of 33m
   Let CP of each metre be Re.1 , Then, CP of 22m= Rs.22, SP of 22m=Rs.33.
   Gain%=[(11/22)*100]% =50%

Ex10. A vendor bought bananas at 6 for Rs.10 and sold them at Rs.4 for Rs.6. Find his gain or loss percent.
   Sol:
   Suppose , number of bananas bought = LCM of 6 and 4=12
   CP =Rs.[(10/6)*12]=Rs.20 ; SP= Rs.[(6/4)*12]=Rs.18
   Loss%=[(2/20)*100]% =10%

Ex11. A man brought toffees at the rate of Re.1. How many for a rupee must he sell to gain 50%?
   Sol. C.P of 3 toffees =Re 1; S.P of 3 toffees =150% of Re.1 =3/2.
   For Rs.3/2, toffees sold =3, for Re.1, toffees sold = [3*(2/3)] = 2.

Ex12. A grocer purchased 80 kg of sugar at Rs.13.50 per kg and mixed it with 120 kg sugar at Rs.16 per kg. At what rate should he sell the mixture to gain 16%?
   Sol. C.P of 200 kg of mixture = Rs. (80 * 13.50+120*16) = Rs.3000.
   S.P =116% of Rs.3000 =Rs.[(116/200) *3000]=Rs.3480.
   Rate of S.P of the mixture =Rs.[3480/200] per kg =Rs.17.40 per kg.

Ex13. Pure ghee cost Rs.100 per kg. After adulterating it with vegetable oil costing Rs.50 per kg, A shopkeeper sells the mixture at the rate of Rs.96 per kg, thereby making a profit of 20%. In what ratio does he mix the two?
   Sol. Mean cost price =Rs. [(100/120)*96 ] =Rs.80 per kg.
By the rate of allegation:

<table>
<thead>
<tr>
<th>C.P of 1kg ghee</th>
<th>C.P of 1kg oil</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>50</td>
</tr>
</tbody>
</table>

Mean price: 80

30

20

Required ratio = 30:20 = 3:2.

Ex. 14. A dishonest dealer professes to sell his goods at cost price but uses a weight of 960 gms for a kg weight. Find his gain percent.

**Sol.**

\[ \text{Gain\%} = \left( \frac{\text{Error}}{\text{error value}} \right) \times 100 = \left( \frac{40}{960} \right) \times 100 = 4\frac{1}{6}\% \]

Ex 15. If the manufacturer gains 10\%, the wholesale dealer 15\% and the retailer 25\% , then find the cost of production of a table, the retail price of which is Rs.1265?

**Sol:**

Let the cost of production of the table be Rs \( x \)

The \( 125\% \) of \( 115\% \) of \( 110\% \) of \( x = 1265 \)

\[ \Rightarrow \frac{125}{100} \times \frac{115}{100} \times \frac{110}{100} \times x = 1265 \Rightarrow \frac{253}{160} \times x = 1265 \Rightarrow x = \frac{1265 \times 160}{253} = Rs. 800 \]

Ex 16. Monika purchased a pressure cooker at \( \frac{9}{10} \) th of its selling price and sold it at 8\% more than its S.P. Find her gain percent.

**Sol:**

Let the S.P be Rs. \( x \), then C.P = Rs. \( \frac{9}{10} \) of Rs. \( x \) = Rs. \( \frac{27x}{25} \)

Gain = Rs. \( \frac{27x}{25} \times \frac{9}{10} \times \frac{108x}{100} = Rs. (108x - 90x)/100 = Rs. 18x/100 \)

Gain\% = \( \left( \frac{18x}{100} \times \frac{10}{9x} \right) \times 100\% = 20\% \)

Ex.17 An article is sold at certain price. By selling it at 2/3 of its price one lossess 10\%, find the gain at original price?

**Sol:**

Let the original S.P be Rs. \( x \). then now S.P = Rs. \( \frac{2x}{3} \), loss = 10\%

now C.P = Rs. \( \frac{20x}{27} \), loss = 35\%

Ex 18. A tradesman sold an article at a loss of 20\%. If the selling price has been increased by Rs 100, there would have been a gain of 5\%. What was the cost price of the article?
Let C.P be Rs. x. then (105% of x) - (80% of x) = 100 or 25% of x = 100

⇒ x/4 = 100 or x = 400

⇒ so, C.P = Rs. 400

Ex 19. A man sells an article at a profit of 25% if he had bought it 20% less and sold it for Rs. 10.50 less, he would have gained 30%. Find the cost price of the article.

Sol:

Let the C.P be Rs. x

1st S.P = 125% of x = 125x/100 = 5x/4; 2nd S.P = 80% of x = 80x/100 = 4x/5

2nd S.P = 130% of 4x/5 = (130/100 * 4x/5) = 26x/25

⇒ 5x/4 - 26x/25 = 10.50

⇒ x = (10.50 * 100) / 21 = 50

hence C.P = Rs. 50

Ex 20. The price of the jewel, passing through three hands, rises on the whole by 65%. If the first and the second sellers 20% and 25% profit respectively find the percentage profit earned by the third seller.

Sol:

Let the original price of the jewel be Rs. p and let the profit earned by the third seller be x%.

Then, (100 + x)% of 125% of 120% of P = 165% of P

⇒ (100 + X)/(100 * 125/100 * 120/100) * P = (165/100) * P

⇒ (100 + X)/(165 * 100 * 100)/(125 * 120) = 110

⇒ X = 10%

Ex 21. A man 2 flats for Rs 675958 each. On one he gains 16% while on the other he losses 16%. How much does he gain/loss in the whole transaction?

Sol:

In this case there will be always loss. The selling price is immaterial.

Hence, loss % = (common loss and gain%)² / 10 = (16/10)% = (64/25)% = 2.56%
Ex.22. A dealer sold three-fourth of his article at a gain of 20% and remaining at a cost price. Find the gain earned by him at the two transaction.

Sol:

Let the C.P of the whole be Rs x

C.P of 3/4 = Rs 3x/4, C.P of 1/4 = Rs x/4

⇒ total S.P = Rs [(120% of 3x/4) + x/4] = Rs (9x/10 + x/4) = Rs 23x/20

⇒ gain = Rs (23x/20 - x) = Rs 3x/20

⇒ gain% = 3x/20 * 1/x * 100)% = 15%

Ex 23. A man bought a horse and a carriage for Rs 3000. He sold the horse at a gain of 20% and the carriage at a loss of 10%, thereby gaining 2% on the whole. Find the cost of the horse.

Sol:

Let the C.P of the horse be Rs x, then C.P of the carriage = Rs (3000 - x)

20% of x - 10% of (3000 - x) = 2% of 3000

⇒ x/5 - (3000 - x)/10 = 60 = .2x - 3000 + x = 600 = .3x + 3600 => x = 1200

⇒ hence, C.P of the horse = Rs 1200

Ex.24. Find the single discount equivalent to a series discount of 20%, 10% and 5%.

Sol:

Let the marked price be Rs 100

then, net S.P = 95% of 90% of 80% of Rs 100

= Rs (95/100 * 90/100 * 80/100 * 100) = Rs 68.40
Ex .25 After getting 2 successive discounts, a shirt with a list price of Rs 150 is available at Rs 105. If the second discount is 12.5%, find the first discount.

Sol:

Let the first discount be \( x\% \)

Then, \( 87.5\% \) of \((100-x)\%\) of 150 = 105

\[ \Rightarrow \frac{87.5}{100} \times \frac{100-x}{100} \times 150 = 105 \Rightarrow 100-x = \frac{(105 \times 100 \times 100)}{(150 \times 87.5)} = 80 \]

\[ \Rightarrow x = (100-80) = 20 \]

\[ \Rightarrow \text{first discount} = 20\% \]

Ex .26 An uneducated retailer marks all its goods at 50% above the cost price and thinking that he will still make 25% profit, offers a discount of 25% on the marked price. What is the actual profit on the sales?

Sol:

Let C.P = Rs 100 then, marked price = Rs 100

S.P = 75% of Rs 150 = Rs 112.50

Hence, gain% = 12.50%

Ex27. A retailer buys 40 pens at the market price of 36 pens from a wholesaler, if he sells these pens giving a discount of 1%, what is the profit %?

Sol:

Let the market price of each pen be Rs 1

then, C.P of 40 pens = Rs 36 S.P of 40 pens = 99% of Rs 40 = Rs 39.60

profit % = \( \frac{(3.60 \times 100)}{36} \% = 10\% \)
Ex 28. At what % above C.P must an article be marked so as to gain 33% after allowing a customer a discount of 5%?

Sol

Let C.P be Rs 100 then S.P be Rs 133

Let the market price be Rs x

Then 90% of x = 133 => 95x/100 = 133 => x = (133 * 100 / 95) = 140

Market price = 40% above C.P

Ex 29. When a producer allows 36% commission on retail price of his product, he earns a profit of 8.8%. What would be his profit % if the commission is reduced by 24%?

Sol:

Let the retail price = Rs 100 then, commission = Rs 36

S.P = Rs (100 - 36) = Rs 64

But, profit = 8.8%

C.P = Rs (100 / 108.8 * 64) = Rs 1000 / 17

New commission = Rs 12. New S.P = Rs (100 - 12) Rs 88

Gain = Rs (88 - 1000 / 17) = Rs 496 / 17

Gain % = (496 / 17 * 17 / 1000 * 100) % = 49.6%
12. RATIO AND PROPORTION

IMPORTANT FACTS AND FORMULAE

I. RATIO: The ratio of two quantities $a$ and $b$ in the same units, is the fraction $a/b$ and we write it as $a:b$. In the ratio $a:b$, we call $a$ as the first term or antecedent and $b$, the second term or consequent.

Ex. The ratio $5:9$ represents $5/9$ with antecedent = 5, consequent = 9.

Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Ex. $4:5 = 8:10 = 12:15$ etc. Also, $4:6 = 2:3$.

2. PROPORTION: \textit{The equality of two ratios is called proportion.}

If $a:b = c:d$, we write, $a:b::c:d$ and we say that $a$, $b$, $c$, $d$ are in proportion. Here $a$ and $d$ are called extremes, while $b$ and $c$ are called mean terms.

Product of means = Product of extremes.

Thus, $a:b::c:d \Leftrightarrow (b \times c) = (a \times d)$.

3. (i) Fourth Proportional: If $a:b = c:d$, then $d$ is called the fourth proportional to $a$, $b$, $c$.

(ii) Third Proportional: If $a:b = b:c$, then $c$ is called the third proportional to $a$ and $b$.

(iii) Mean Proportional: Mean proportional between $a$ and $b$ is square root of $ab$.

4. (i) COMPARISON OF RATIOS:

We say that $(a:b) > (c:d) \Leftrightarrow (a/b) > (c/d)$.

(ii) COMPOUNDED RATIO:

The compounded ratio of the ratios $(a:b)$, $(c:d)$, $(e:f)$ is $(ace:bdf)$.

5. (i) \textit{Duplicate ratio} of $(a:b)$ is $(a^2:b^2)$.

(ii) \textit{Sub-duplicate ratio} of $(a:b)$ is $(\sqrt{a} : \sqrt{b})$.

(iii) \textit{Triplet ratio} of $(a:b)$ is $(a^3 : b^3)$.

(iv) \textit{Sub-triplet ratio} of $(a:b)$ is $(a^{1/3} : b^{1/3})$.

(v) If $(a/b) = (c/d)$, then $((a+b)/(a-b)) = ((c+d)/(c-d))$ \textit{(Componendo and dividendo)}.

6. VARIATION:

(i) We say that $x$ is directly proportional to $y$, if $x = ky$ for some constant $k$ and we write, $x \propto y$.

(ii) We say that $x$ is inversely proportional to $y$, if $xy = k$ for some constant $k$ and we write, $x \propto (1/y)$.
SOLVED PROBLEMS

Ex. 1. If \( a : b = 5 : 9 \) and \( b : c = 4 : 7 \), find \( a : b : c \).

Sol. \( a:b=5:9 \) and \( b:c=4:7 \)
\[ a:b:c = 5:9:63/4 = 20:36:63. \]

Ex. 2. Find:
   (i) the fourth proportional to 4, 9, 12;
   (ii) the third proportional to 16 and 36;
   (iii) the mean proportional between 0.08 and 0.18.

Sol.
   (i) Let the fourth proportional to 4, 9, 12 be \( x \).
      Then, \( 4 : 9 : : 12 : x \) \( \Leftrightarrow \) \( 4 \times x = 9 \times 12 \) \( \Leftrightarrow \) \( x = (9 \times 12)/14 = 27; \)
      Fourth proportional to 4, 9, 12 is 27.

   (ii) Let the third proportional to 16 and 36 be \( x \).
      Then, \( 16 : 36 : : 36 : x \) \( \Leftrightarrow \) \( 16 \times x = 36 \times 36 \) \( \Leftrightarrow \) \( x = (36 \times 36)/16 = 81 \)
      Third proportional to 16 and 36 is 81.

   (iii) Mean proportional between 0.08 and 0.18
      \( \sqrt{0.08 \times 0.18} = \sqrt{8/100 \times 18/100} = \sqrt{144/(100 \times 100)} = 12/100 = 0.12 \)

Ex. 3. If \( x : y = 3 : 4 \), find \( (4x + 5y) : (5x - 2y) \).

Sol. \( X/Y=3/4 \) \( \Leftrightarrow \) \( (4x+5y)/(5x+2y) = (4( x/y)+5)/(5 (x/y)-2) = (4(3/4)+5)/(5(3/4)-2) \)
\[ =(3+5)/(7/4) = 32/7 \]

Ex. 4. Divide Rs. 672 in the ratio 5 : 3.

Sol. Sum of ratio terms = (5 + 3) = 8.
First part = Rs. \((672 \times (5/8)) = Rs. 420; \) Second part = Rs. \((672 \times (3/8)) = Rs. 252. \)
Ex. 5. **Divide Rs. 1162 among A, B, C in the ratio 35 : 28 : 20.**

**Sol.** Sum of ratio terms = (35 + 28 + 20) = 83.

A's share = Rs. (1162 x (35/83)) = Rs. 490; B's share = Rs. (1162 x (28/83)) = Rs. 392;

C's share = Rs. (1162 x (20/83)) = Rs. 280.

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Ex. 6. **A bag contains 50 p, 25 P and 10 p coins in the ratio 5: 9: 4, amounting to Rs. 206. Find the number of coins of each type.**

**Sol.** Let the number of 50 p, 25 p and 10 p coins be 5x, 9x and 4x respectively.

\[
\frac{5x}{2} + \frac{9x}{4} + \frac{4x}{10} = 206 \iff 50x + 45x + 8x = 4120 \iff 103x = 4120 \iff x = 40.
\]

Number of 50 p coins = (5 x 40) = 200; Number of 25 p coins = (9 x 40) = 360;

Number of 10 p coins = (4 x 40) = 160.

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Ex. 7. **A mixture contains alcohol and water in the ratio 4 : 3. If 5 litres of water is added to the mixture, the ratio becomes 4: 5. Find the quantity of alcohol in the given mixture.**

**Sol.** Let the quantity of alcohol and water be 4x litres and 3x litres respectively.

\[
\frac{4x}{3x+5} = \frac{4}{5} \iff 20x = 4(3x+5) \iff 8x = 20 \iff x = 2.5
\]

Quantity of alcohol = (4 x 2.5) litres = 10 litres.
13. PARTNERSHIP

IMPORTANT FACTS AND FORMULA

1. Partnership: When two or more than two persons run a business jointly, they are called partners and the deal is known as partnership.

2. Ratio of Division of Gains:
   i) When investments of all the partners are for the same time, the gain or loss is distributed among the partners in the ratio of their investments.
   Suppose A and B invest Rs. x and Rs. y respectively for a year in a business, then at the end of the year:
   
   \[
   \frac{A's \ share \ of \ profit}{B's \ share \ of \ profit} = \frac{x}{y}.
   \]

   ii) When investments are for different time periods, then equivalent capitals are calculated for a unit of time by taking (capital \times number \ of \ units \ of \ time). Now, gain or loss is divided in the ratio of these capitals.
   Suppose A invests Rs. x for p months and B invests Rs. y for q months, then
   
   \[
   \frac{A's \ share \ of \ profit}{B's \ share \ of \ profit} = \frac{xp}{yq}.
   \]

3. Working and Sleeping Partners: A partner who manages the business is known as a working partner and the one who simply invests the money is a sleeping partner.

SOLVED EXAMPLES

Ex. 1. A, B and C started a business by investing Rs. 1,20,000, Rs. 1,35,000 and Rs.1,50,000 respectively. Find the share of each, out of an annual profit of Rs. 56,700.

Sol. Ratio of shares of A, Band C = Ratio of their investments

\[
\frac{120000}{135000} : \frac{150000}{150000} = 8 : 9 : 10.
\]

A’s share = Rs. \((56700 \times 8/27)\) = Rs. 16800.

B's share = Rs. \((56700 \times 9/27)\) = Rs. 18900.

C’s share = Rs. \((56700 \times 10/27)\) = Rs. 21000.

Ex. 2. Alfred started a business investing Rs. 45,000. After 3 months, Peter joined him with a capital of Rs. 60,000. After another 6 months, Ronald joined them with a capital of Rs. 90,000. At the end of the year, they made a profit of Rs. 16,500. Find
the lire of each.

**Sol.** Clearly, Alfred invested his capital for 12 months, Peter for 9 months and Ronald for 3 months.

So, ratio of their capitals = \((45000 \times 12) : (60000 \times 9) : (90000 \times 3)\)

\[= 540000 : 540000 : 270000 = 2 : 2 : 1.\]

Alfred's share = Rs. \((16500 \times (2/5)) = Rs. 6600\)

Peter's share = Rs. \((16500 \times (2/5)) = Rs. 6600\)

Ronald's share = Rs. \((16500 \times (1/5)) = Rs. 3300.\)

**Ex. 3.** A, Band C start a business each investing Rs. 20,000. After 5 months A withdrew Rs.6000 B withdrew Rs. 4000 and C invests Rs. 6000 more. At the end of the year, a total profit of Rs. 69,900 was recorded. Find the share of each.

**Sol.** Ratio of the capitals of A, Band C

\[= 20000 \times 5 + 15000 \times 7 : 20000 \times 5 + 16000 \times 7 : 20000 \times 5 + 26000 \times 7\]

\[= 205000 : 212000 : 282000 = 205 : 212 : 282.\]

A's share = Rs. \(69900 \times (205/699) = Rs. 20500\) \[I\]

B's share = Rs. \(69900 \times (212/699) = Rs. 21200;\)

C's share = Rs. \(69900 \times (282/699) = Rs. 28200.\)

**Ex. 4.** A, Band C enter into partnership. A invests 3 times as much as B and B invests two-third of what C invests. At the end of the year, the profit earned is Rs. 6600. What is the share of B ?

**Sol.** Let C's capital = Rs. \(x\). Then, B's capital = Rs. \((2/3)x\)

\[A's \text{ capital} = Rs. \left(3 \times \frac{2}{3} \times x\right) = Rs. \ 2x.\]

Ratio of their capitals = \(2x : (2/3)x = 6 : 2 : 3.\)

Hence, B's share = Rs. \((6600 \times (2/11)) = Rs. 1200.\)

**Ex. 5.** Four milkmen rented a pasture. A grazed 24 cows for 3 months; B 10 for 5 months; C 35 cows for 4 months and D 21 cows for 3 months. If A's share of rent is Rs. 720, find the total rent of the field.

**Sol.** Ratio of shares of A, B, C, D = \((24 \times 3) : (10 \times 5) : (35 \times 4) : (21 \times 3) = 72 : 50 : 140\)
Let total rent be Rs. x. Then, A’s share = Rs. \((\frac{72x}{325})\)

\((\frac{72x}{325}) = 720 \Leftrightarrow x = (720 \times 325)/72 = 3250\)

Hence, total rent of the field is Rs. 3250.

**Ex.6.** A invested Rs. 76,000 in a business. After few months, B joined him Rs. 57,000. At the end of the year, the total profit was divided between them in ratio 2 : 1. After how many months did B join?

Sol. Suppose B joined after x months. Then, B's money was invested for \((12 - x)\)

\((\frac{76000 \times 12}{57000 \times (12-x)}) = 2/1 \Leftrightarrow 912000 = 114000(12-x)\)

114 \((12 - x) = 912 \Leftrightarrow 12 - x = 8 \Leftrightarrow x = 4\)

Hence, B joined after 4 months.

**Ex.7.** A, B, and C enter into a partnership by investing in the ratio of 3 : 2 : 4. After 1 year, B invests another Rs. 2,70,000 and C, at the end of 2 years, also invests Rs.2,70,000. At the end of three years, profits are shared in the ratio of 3 : 4 : 5. Find initial investment of each.

Sol. Let the initial investments of A, B, and C be Rs. \(3x\), Rs. \(2x\) and Rs. \(4x\) respectively. Then,

\(3x \times 36 : [(2x \times 12) + (2x + 270000) \times 24] : [(4x \times 24) + (4x + 270000) \times 12] = 3 : 4 : 5\)

\(108x : (72x + 6480000) : (144x + 32400000) = 3 : 4 : 5\)

\(108x/(72x+6480000) = 3/4 \Leftrightarrow 432x = 216x + 19440000\)

\(216x = 19440000\)

\(x = 90000\)

Hence, A’s initial investment = 3x = Rs. 2,70,000;
B’s initial investment = 2x = Rs. 1,80,000;
C’s initial investment = 4x = Rs. 3,60,000.
14. CHAIN RULE

IMPORTANT FACTS AND FORMULA

1. Direct Proportion: Two quantities are said to be directly proportional, if on the increase (or decrease) of the one, the other increases (or decreases) to the same extent.
   Ex. 1. Cost is directly proportional to the number of articles.
   (More Articles, More Cost)
   Ex. 2. Work done is directly proportional to the number of men working on it.
   (More Men, More Work)

2. Indirect Proportion: Two quantities are said to be indirectly proportional, if on the increase of the one, the other decreases to the same extent and vice-versa.
   Ex. 1. The time taken by a car in covering a certain distance is inversely proportional to the speed of the car.
   (More speed, Less is the time taken to cover a distance)
   Ex. 2. Time taken to finish a work is inversely proportional to the number of persons working at it.
   (More persons, Less is the time taken to finish a job)

Remark: In solving questions by chain rule, we compare every item with the term to be found out.

SOLVED EXAMPLES

Ex. 1. If 15 toys cost Rs. 234, what do 35 toys cost?
Sol. Let the required cost be Rs. x. Then,
   More toys, More cost (Direct Proportion)
   \[15 : 35 : : 234 : x \Rightarrow (15 \times x) = (35 \times 234) \Rightarrow x = (35 \times 234)/15 = 546\]
   Hence, the cost of 35 toys is Rs. 546.

Ex. 2. If 36 men can do a piece of work in 25 hours, in how many hours will 15 men do it?
Sol. Let the required number of hours be x. Then,
   Less men, More hours (Indirect Proportion)
   \[15 : 36 : : 25 : x \Rightarrow (15 \times x) = (36 \times 25) \Rightarrow (36 \times 25)/15 = 60\]
   Hence, 15 men can do it in 60 hours.
Ex. 3. If the wages of 6 men for 15 days be Rs.2100, then find the wages of for 12 days.

Sol. Let the required wages be Rs. x.

More men, More wages (Direct Proportion)

Less days, Less wages (Direct Proportion)

Men 6: 9 : : 2100: x

Days 15: 12

Therefore (6 x 15 x x) = (9 x 12 x 2100) ⇔ x = (9 x 12 x 2100) / (6 x 15) = 2520

Hence the required wages are Rs. 2520.

Ex. 4. If 20 men can build a wall 66 metres long in 6 days, what length of a similar can be built by 86 men in 8 days?

Sol. Let the required length be x metres

More men, More length built (Direct Proportion)

Less days, Less length built (Direct Proportion)

Men 20: 35

Days 6: 3 : : 56 : x

Therefore (20 x 6 x x) = (35 x 3 x 56) ⇔ x = (35 x 3 x 56) / 120 = 49

Hence, the required length is 49 m.

Ex. 5. If 15 men, working 9 hours a day, can reap a field in 16 days, in how many days will 18 men reap the field, working 8 hours a day?

Sol. Let the required number of days be x.

More men, Less days (indirect proportion)

Less hours per day, More days (indirect proportion)

Men 18: 15

Hours per day 8: 9 : : 16 : x

(18 x 8 x x) = (15 x 9 x 16) ⇔ x = (44 x 15) / 144 = 15

Hence, required number of days = 15.

Ex. 6. If 9 engines consume 24 metric tonnes of coal, when each is working 8 hours
day, bow much coal will be required for 8 engines, each running 13 hours a day, it being given that 3 engines of former type consume as much as 4 engines of latter type?

Sol. Let 3 engines of former type consume 1 unit in 1 hour.
Then, 4 engines of latter type consume 1 unit in 1 hour.
Therefore 1 engine of former type consumes $(1/3)$ unit in 1 hour.
1 engine of latter type consumes $(1/4)$ unit in 1 hour.
Let the required consumption of coal be $x$ units.
Less engines, Less coal consumed (direct proportion)
More working hours, More coal consumed (direct proportion)
Less rate of consumption, Less coal consumed (direct proportion)
Number of engines $9: 8$
Working hours $8 : 13 \{24 : x$
Rate of consumption $(1/3):(1/4)$
\[
[ 9 \times 8 \times (1/3) \times x) = (8 \times 13 \times (1/4) \times 24 ) \Leftrightarrow 24x = 624 \Leftrightarrow x = 26.
\]
Hence, the required consumption of coal = 26 metric tonnes.

Ex. 7. A contract is to be completed in 46 days sad 117 men were said to work 8 hours a day. After 33 days, $(4/7)$ of the work is completed. How many additional men may be employed so that the work may be completed in time, each man now working 9 hours a day?

Sol. Remaining work = $(1-(4/7))=(3/7)$
Remaining period = $(46 - 33)$ days = 13 days
Let the total men working at it be $x$.
Less work, Less men (Direct Proportion)
Less days, More men (Indirect Proportion)
More Hours per Day, Less men (Indirect Proportion)
\[
\text{Work} = (4/7):(3/7)
\text{Days} = 13:33 \quad \{117: x
\text{Hrs/day} = 9 : 8
\text{Therefore} \quad (4/7) \times 13 \times 9 \times x = (3/7) \times 33 \times 8 \times 117 \text{ or } x = (3 \times 33 \times 8 \times 117) / (4 \times 13 \times 9) = 198
\]
Additional men to be employed = $(198 - 117) = 81$.

Ex. 8. A garrison of 3300 men had provisions for 32 days, when given at the rate of 860 gns per head. At the end of 7 days, a reinforcement arrives and it was for that the provisions will last 17 days more, when given at the rate of 826 gms per head, What is the strength of the reinforcement?
Sol. The problem becomes:

3300 men taking 850 gms per head have provisions for (32 - 7) or 25 days,
How many men taking 825 gms each have provisions for 17 days?

Less ration per head, more men  (Indirect Proportion)
Less days, More men  (Indirect Proportion)

Ration 825 : 850
Days 17: 25  } : : 3300 : x

\[(825 \times 17 \times x) = 850 \times 25 \times 3300\] or \[x = (850 \times 25 \times 3300)/(825 \times 17) = 5000\]

Strength of reinforcement = (5500 - 3300) = 1700.
15. TIME AND WORK

IMPORTANT FACTS AND FORMULA

1. If A can do a piece of work in n days, then A's 1 day's work = (1/n).

2. If A’s 1 day's work = (1/n), then A can finish the work in n days.

3. A is thrice as good a workman as B, then:
   Ratio of work done by A and B = 3 : 1.
   Ratio of times taken by A and B to finish a work = 1 : 3.

SOLVED EXAMPLES

Ex. 1. Worker A takes 8 hours to do a job. Worker B takes 10 hours to do the same job. How long should it take both A and B, working together but independently, to do the same job? (IGNOU, 2003)

Sol. A’s 1 hour's work = 1/8

   B's 1 hour's work = 1/10

   (A + B)'s 1 hour's work = (1/8) + (1/10) = 9/40

   Both A and B will finish the work in 40/9 days.

Ex. 2. A and B together can complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can B alone complete that work? (Bank P.O. 2003)

Sol. (A + B)'s 1 day's work = (1/4). A's 1 day's work = (1/12).

   B's 1 day's work = ((1/4) - (1/12)) = (1/6)

   Hence, B alone can complete the work in 6 days.

Ex. 3. A can do a piece of work in 7 days of 9 hours each and B can do it in 6 days
of 7 hours each. How long will they take to do it, working together 8 hours a day?

**Sol.** A can complete the work in \((7 \times 9) = 63\) hours.
B can complete the work in \((6 \times 7) = 42\) hours.

A’s 1 hour's work = \((1/63)\) and B's 1 hour's work =\((1/42)\)

\((A + B)'s\) 1 hour's work =\((1/63)+(1/42)=(5/126)\)

Both will finish the work in \((126/5)\) hrs.
Number of days of \((42/5)\) hrs each =\((126 \times 5)/(5 \times 42)=3\) days

**Ex. 4.** A and B can do a piece of work in 18 days; Band C can do it in 24 days A and C can do it in 36 days. In how many days will A, Band C finish it together and separately?

**Sol.** \((A + B)'s\) 1 day's work = \((1/18)\) \((B + C)'s\) 1 day's work = \((1/24)\)
and \((A + C)'s\) 1 day's work = \((1/36)\)

Adding, we get: 2 \((A + B + C)'s\) 1 day's work =\((1/18 + 1/24 + 1/36)\)

\(=9/72 =1/8\)

\((A +B + C)'s\) 1 day's work =\((1/8)\)

Thus, A, Band C together can finish the work in 16 days.
Now, A’s 1 day's work = \([(A + B + C)'s\) 1 day's work] - \([(B + C)'s\) 1 day work:
\(=(1/16 − 1/24)= 1/48\)

A alone can finish the work in 48 days.
Similarly, B's 1 day's work =\((1/16 − 1/36)=5/144\)

B alone can finish the work in \(144/5=28\ 4/5\) days

And C’s 1 day work =\((1/16-1/18)=1/144\)

Hence C alone can finish the work in 144 days.

**Ex. 6.** A is twice as good a workman as B and together they finish a piece in 18 days. In how many days will A alone finish the work?

**Sol.** \((A’s\) 1 day’s work):\((B’s\) 1 days work) = 2 : 1.

\((A + B)'s\) 1 day's work = \((1/18)\)

\(\text{Click here to purchase a license to embed this image}\)
Divide \(\frac{1}{18}\) in the ratio 2 : 1.

\[\therefore \text{A's 1 day's work} = \left(\frac{1}{18} \times \frac{2}{3}\right) = \frac{1}{27}\]

Hence, A alone can finish the work in 27 days.

**Ex. 6.** A can do a certain job in 12 days. B is 60% more efficient than A. How many days does B alone take to do the same job?

**Sol.** Ratio of times taken by A and B = 160 : 100 = 8 : 5.

Suppose B alone takes \(x\) days to do the job.

Then, 8 : 5 :: 12 : \(x\) = \(8x = 5 \times 12 = x = 7\ 1/2\) days.

**Ex. 7.** A can do a piece of work in 80 days. He works at it for 10 days B alone finishes the remaining work in 42 days. In how much time will A and B working together, finish the work?

**Sol.** Work done by A in 10 days = \(\left(\frac{1}{80} \times 10\right) = \frac{1}{8}\)

Remaining work = \(1 - \frac{1}{8}\) = \(\frac{7}{8}\)

Now, \(\frac{7}{8}\) work is done by B in \(42 \times \frac{8}{7}\) = 48 days.

A's 1 day's work = \(\frac{1}{80}\) and B's 1 day's work = \(\frac{1}{48}\)

\((A+B)\)'s 1 day's work = \(\left(\frac{1}{80} + \frac{1}{48}\right) = \frac{8}{240} = \frac{1}{30}\)

Hence, both will finish the work in 30 days.

**Ex. 8.** A and B undertake to do a piece of work for Rs. 600. A alone can do it in 6 days while B alone can do it in 8 days. With the help of C, they finish it in 3 days. Find the share of each.

**Sol :** C's 1 day's work = \(\frac{1}{3} - (\frac{1}{6} + \frac{1}{8}) = \frac{24}{24} = \frac{1}{2}\)

\(A : B : C = \text{Ratio of their 1 day's work} = 1/6:1/8:1/24 = 4 : 3 : 1.\)

A's share = Rs. \((600 \times \frac{4}{8}) = Rs.300\), B's share = Rs. \((600 \times \frac{3}{8}) = Rs. 225.\)

C's share = Rs. \((600 - (300 + 225)) = Rs. 75.\)

**Ex. 9.** A and B working separately can do a piece of work in 9 and 12 days respectively. If they work for a day alternately, A beginning, in how many days, the work will be completed?

\((A + B)\)'s 2 days' work = \(\left(\frac{1}{9} + \frac{1}{12}\right) = \frac{7}{36}\)

Work done in 5 pairs of days = \(\left(5\times\frac{7}{36}\right) = \frac{35}{36}\)

Remaining work = \(\left(1 - \frac{35}{36}\right) = \frac{1}{36}\)
On 11th day, it is A’s turn. 1/9 work is done by him in 1 day.

1/36 work is done by him in(9*1/36)=1/4 day
Total time taken = (10 + 1/4) days = 10 1/4 days.

Ex 10. 45 men can complete a work in 16 days. Six days after they started working, 30 more men joined them. How many days will they now take to complete the remaining work?

(45 x 16) men can complete the work in 1 day.

1 man's 1 day's work = 1/720

45 men's 6 days' work = (1/16*6)=3/8

Remaining work = (1-3/8)=5/8

75 men's 1 day's work = 75/720=5/48

Now, 5 work is done by them in 1 day.

5 work is done by them in (48 x 5)=6 days.

Ex: 11. 2 men and 3 boys can do a piece of work in 10 days while 3 men and 2 boys can do the same work in 8 days. In how many days can 2 men and 1 boy do the work?

Soln: Let 1 man’s 1 day’s work = x and 1 boy’s 1 day’s work = y.

Then, 2x + 3y = 1 and 3x + 2y = 1

Solving, we get: x = 7/200 and y = 1/100

(2 men + 1 boy)’s 1 day’s work = (2 x 7/200 + 1 x 1/100) = 16/200 = 2/25

So, 2 men and 1 boy together can finish the work in 25/2 = 12 1/2 days.
16. PIPES AND CISTERNS

IMPORTANT FACTS AND FORMULAE

1. **Inlet**: A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.

**Outlet**: A pipe connected with a tank or a cistern or a reservoir, emptying it, is known as an outlet.

2. (i) If a pipe can fill a tank in \(x\) hours, then: part filled in 1 hour = \(\frac{1}{x}\)

(ii) If a pipe can empty a full tank in \(y\) hours, then: part emptied in 1 hour = \(\frac{1}{y}\)

(iii) If a pipe can fill a tank in \(x\) hours and another pipe can empty the full tank in \(y\) hours (where \(y > x\)), then on opening both the pipes, the net part filled in 1 hour = \(\frac{1}{x} - \frac{1}{y}\)

(iv) If a pipe can fill a tank in \(x\) hours and another pipe can empty the full tank in \(y\) hours (where \(x > y\)), then on opening both the pipes, the net part emptied in 1 hour = \(\frac{1}{y} - \frac{1}{x}\)

SOLVED EXAMPLES

**Ex. 1**: Two pipes A and B can fill a tank in 36 hours and 46 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

**Sol**: Part filled by A in 1 hour = \(\frac{1}{36}\);

Part filled by B in 1 hour = \(\frac{1}{45}\);

Part filled by (A + B) in 1 hour = \(\frac{1}{36} + \frac{1}{45} = \frac{9}{180} = \frac{1}{20}\)

Hence, both the pipes together will fill the tank in 20 hours.

**Ex. 2**: Two pipes can fill a tank in 10 hours and 12 hours respectively while a third, pipe empties the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time will the tank be filled?

**Sol**: Net part filled in 1 hour = \(\frac{1}{10} + \frac{1}{12} - \frac{1}{20} = \frac{8}{60} = \frac{2}{15}\).

The tank will be full in \(\frac{15}{2}\) hrs = 7 hrs 30 min.
Ex. 3: If two pipes function simultaneously, the reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does it take the second pipe to fill the reservoir?

\[ \text{Sol: let the reservoir be filled by first pipe in} \ x \ \text{hours.} \]

Then, second pipe fill it in \( (x+10) \text{ hrs.} \)

Therefore \( \frac{1}{x} + \frac{1}{x+10} = \frac{1}{12} \)
\[ \Leftrightarrow x(x+10) = (x+10+x)(x)(x+10) = (1/12) \]
\[ \Leftrightarrow x^2 -14x -120 = 0 \]
\[ \Leftrightarrow (x-20)(x+6) = 0 \]
\[ \Leftrightarrow x=20 \quad \text{[neglecting the negative value of} \ x] \]

so, the second pipe will take \( (20+10) \text{ hrs.} \) (i.e) 30 hours to fill the reservoir

Ex. 4: A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the 3 are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern?

\[ \text{Sol: Work done by the waste pipe in} \ 1 \text{min} \]

\[ = (1/20) - (1/12) + (1/15) = -1/10 \quad \text{[negative sign means emptying]} \]

therefore the waste pipe will empty the full cistern in 10 min

Ex. 5: An electric pump can fill a tank in 3 hours. Because of a leak in, the tank it took \( 3\frac{1}{2} \) hours to fill the tank. If the tank is full, how much time will the leak take to empty it?

\[ \text{Sol: work done by the leak in} \ 1 \text{hr} = (1/3) - (1/(7/2)) = (1/3) - (2/7) = (1/21). \]

The leak will empty the tank in 21 hours.

Ex. 6. Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it took 32 minutes more to fill the cistern. When the cistern is full, in what time will the leak empty it?

\[ \text{Sol: Work done by the two pipes in} \ 1 \text{hr} = (1/14) + (1/16) = (15/112). \]
Time taken by these pipes to fill the tank = \((112/15)\) hrs = 7 hrs 28 min.
Due to leakage, time taken = 7 hrs 28 min + 32 min = 8 hrs
Work done by (two pipes + leak) in 1 hour = \((1/8)\).
Work done by the leak in 1 hour =\((15/112)-(1/8)=1/112\).
Leak will empty the full cistern in 112 hours.

Ex. 7: Two pipes A and B can fill a tank in 36 min. and 45 min. respectively. A water pipe C can empty the tank in 30 min. First A and B are opened. after 7 min,C is also opened. In how much time, the tank is full?

**Sol:** Part filled in 7 min. = 7*\(((1/36)+(1/45))=7/20\).
Remaining part=\((1-(7/20))=13/20\).
Net part filled in 1 min. when A,B and C are opened=\((1/36)+(1/45)-(1/30)=1/60\).
Now, \((1/60)\) part is filled in one minute.
\((13/20)\) part is filled in \((60*(13/20))=39 \) minutes.

Ex.8: Two pipes A,B can fill a tank in 24 min. and 32 min. respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is full in 18 min.?

**Sol:** let B be closed after x min. then ,
Part filled by (A+B) in x min. +part filled by A in (18-x)min.=1
Therefore x*\(((1/24)+(1/32))+(18-x)*(1/24)=1 \iff (7x/96) + ((18-x)/24)=1.\)
\iff 7x +4*(18-x)=96.
Hence, B must be closed after 8 min.
17. TIME AND DISTANCE

IMPORTANT FACTS AND FORMULA

1. Speed = \( \frac{\text{Distance}}{\text{Time}} \), \( \text{Time}= \frac{\text{Distance}}{\text{Speed}} \), \( \text{Distance} = (\text{Speed} \times \text{Time}) \)

2. \( x \text{ km/hr} = \frac{x}{18} \times 5 \)

3. \( x \text{ m/sec} = \left( x \times \frac{18}{5}\right) \text{ km/hr} \)

4. If the ratio of the speeds of A and B is \( a:b \), then the ratio of the times taken by them to cover the same distance is \( \frac{1}{a} : \frac{1}{b} \) or \( b:a \).

5. Suppose a man covers a certain distance at \( x \text{ km/hr} \) and an equal distance at \( y \text{ km/hr} \). Then, the average speed during the whole journey is \( \frac{2xy}{x+y} \text{ km/hr} \).

SOLVED EXAMPLES

Ex. 1. How many minutes does Aditya take to cover a distance of 400 m, if he runs at a speed of 20 km/hr?
Sol. Aditya’s speed = 20 km/hr = \( \frac{20 \times 5}{18} \) m/sec = \( \frac{50}{9} \) m/sec
∴ Time taken to cover 400 m = \( \frac{400 \times 9}{50} \) sec = 72 sec = 1 \( \frac{12}{60} \) min = 1 \( \frac{1}{5} \) min.

Ex. 2. A cyclist covers a distance of 750 m in 2 min 30 sec. What is the speed in km/hr of the cyclist?
Sol. Speed = \( \frac{750}{150} \) m/sec = 5 m/sec = \( 5 \times \frac{18}{5} \) km/hr = 18 km/hr

Ex. 3. A dog takes 4 leaps for every 5 leaps of a hare but 3 leaps of a dog are equal to 4 leaps of the hare. Compare their speeds.
Sol. Let the distance covered in 1 leap of the dog be \( x \) and that covered in 1 leap of the hare by \( y \).
Then, \( 3x = 4y \Rightarrow x = \frac{4y}{3} \Rightarrow 4x = \frac{16y}{3} \).
∴ Ratio of speeds of dog and hare = Ratio of distances covered by them in the same time
= \( 4x : 5y = \frac{16}{3} \times \frac{y}{5} = 16:15 \)
Ex. 4. While covering a distance of 24 km, a man noticed that after walking for 1 hour and 40 minutes, the distance covered by him was \( \frac{5}{3} \) of the remaining distance. What was his speed in metres per second?

**Sol.** Let the speed be \( x \) km/hr.

Then, distance covered in 1 hr. 40 min. i.e., \( \frac{2}{3} \) hrs = \( \frac{5x}{3} \) km

Remaining distance = \( \{ 24 - \frac{5x}{3} \} \) km.

\[ \therefore \quad \frac{5x}{3} = \frac{5}{3} \quad \frac{24 - \frac{5x}{3}}{3} \implies \frac{5x}{3} = \frac{5}{3} \quad \frac{72 - 5x}{3} \implies \frac{7x}{3} = 72 - 5x \]

\[ \implies 12x = 72 \implies x = 6 \]

Hence speed = 6 km/hr = \( \{ 6 \times \frac{5}{18} \} \) m/sec = \( \frac{5}{3} \) m/sec = \( \frac{5}{3} \times \frac{18}{3} \)

Ex. 5. Peter can cover a certain distance in 1 hr. 24 min. by covering two-third of the distance at 4 kmph and the rest at 5 kmph. Find the total distance.

**Sol.** Let the total distance be \( x \) km. Then,

\[ \frac{2}{3} x + \frac{1}{3} x = \frac{7}{4} x \implies x + \frac{x}{6} = \frac{7}{5} x \implies 7x = 42 \implies x = 6 \]

Ex. 6. A man traveled from the village to the post-office at the rate of 25 kmph and walked back at the rate of 4 kmph. If the whole journey took 5 hours 48 minutes, find the distance of the post-office from the village.

**Sol.** Average speed = \( \{ \frac{2xy}{x+y} \} \) km/hr = \( \{ \frac{2 \times 25 \times 4}{25 + 4} \} \) km/hr = \( \frac{200}{29} \) km/hr

Distance traveled in 5 hours 48 minutes i.e., \( \frac{24}{5} \) hrs. = \( \{ 200 \times \frac{29}{5} \} \) km = 40 km

Distance of the post-office from the village = \( \{ \frac{40}{2} \} = 20 \) km

Ex. 7. An aeroplane flies along the four sides of a square at the speeds of 200, 400, 600 and 800 km/hr. Find the average speed of the plane around the field.

**Sol.**

Let each side of the square be \( x \) km and let the average speed of the plane around the field by \( y \) km per hour then,

\[ \frac{x}{200} + \frac{x}{400} + \frac{x}{600} + \frac{x}{800} = \frac{4x}{y} \implies 25x/2500 \equiv 4x/y \equiv y = (2400 \times 4/25) = 384 \]

Hence average speed = 384 km/hr

Ex. 8. Walking at \( \frac{5}{7} \) of its usual speed, a train is 10 minutes too late. Find its usual time to cover the journey.

**Sol.** New speed = \( \frac{5}{6} \) of the usual speed
New time taken = \( \frac{6}{5} \) of the usual time
So, \( \left( \frac{6}{5} \text{ of the usual time} \right) - \text{usual time} = 10 \) minutes.
\[ \Rightarrow \text{usual time} = 10 \text{ minutes} \]

**Ex. 9.** If a man walks at the rate of 5 kmph, he misses a train by 7 minutes. However, if he walks at the rate of 6 kmph, he reaches the station 5 minutes before the arrival of the train. Find the distance covered by him to reach the station.

**Sol.** Let the required distance be \( x \) km
Difference in the time taken at two speeds = 1 min = \( \frac{1}{60} \) hr
Hence \( \frac{x}{5} - \frac{x}{6} = \frac{1}{5} \Rightarrow 6x - 5x = 6 \)
\[ \Rightarrow x = 6 \]
Hence, the required distance is 6 km

**Ex. 10.** A and B are two stations 390 km apart. A train starts from A at 10 a.m. and travels towards B at 65 kmph. Another train starts from B at 11 a.m. and travels towards A at 35 kmph. At what time do they meet?

**Sol.** Suppose they meet \( x \) hours after 10 a.m. Then, (Distance moved by first in \( x \) hrs) + [Distance moved by second in \((x-1)\) hrs] = 390.

\[
65x + 35(x-1) = 390 \Rightarrow 100x = 425 \Rightarrow x = \frac{17}{4}
\]

So, they meet 4 hrs.15 min. after 10 a.m i.e., at 2.15 p.m.

**Ex. 11.** A goods train leaves a station at a certain time and at a fixed speed. After \( ^{1/4} \) hours, an express train leaves the same station and moves in the same direction at a uniform speed of 90 kmph. This train catches up the goods train in 4 hours. Find the speed of the goods train.

**Sol.** Let the speed of the goods train be \( x \) kmph.
Distance covered by goods train in 10 hours = Distance covered by express train in 4 hours
\[
10x = 4 \times 90 \text{ or } x = 36.
\]
So, speed of goods train = 36 kmph.

**Ex. 12.** A thief is spotted by a policeman from a distance of 100 metres. When the policeman starts the chase, the thief also starts running. If the speed of the thief be 8 km/hr and that of the policeman 10 km/hr, how far the thief will have run before he is overtaken?

**Sol.** Relative speed of the policeman = \((10 - 8)\) km/hr = 2 km/hr.
Time taken by police man to cover 100m = \( \frac{100}{20} \) hr = 1 hr.

In 1 hrs, the thief covers a distance of \( 8 \times \frac{1}{20} \text{ km} = \frac{2}{5} \text{ km} = 400 \text{ m} \)
Ex.13. I walk a certain distance and ride back taking a total time of 37 minutes. I could walk both ways in 55 minutes. How long would it take me to ride both ways?

Sol. Let the distance be x km. Then,

( Time taken to walk x km) + (time taken to ride x km) = 37 min.
( Time taken to walk 2x km ) + ( time taken to ride 2x km )= 74 min.

But, the time taken to walk 2x km = 55 min.
Time taken to ride 2x km = (74-55) min = 19 min.
18. PROBLEMS ON TRAINS

IMPORTANT FACTS AND FORMULA

1. \( a \) km/hr = \((a \times 5/18)\) m/s.

2. \( a \) m/s = \((a \times 18/5)\) km/hr.

3. Time taken by a train of length \( l \) metres to pass a pole or a standing man or a signal post is equal to the time taken by the train to cover \( l \) metres.

4. Time taken by a train of length \( l \) metres to pass a stationary object of length \( b \) metres is the time taken by the train to cover \((l + b)\) metres.

5. Suppose two trains or two bodies are moving in the same direction at \( u \) m/s and \( v \) m/s, where \( u > v \), then their relative speed = \((u - v)\) m/s.

6. Suppose two trains or two bodies are moving in opposite directions at \( u \) m/s and \( v \) m/s, then their relative speed is = \((u + v)\) m/s.

7. If two trains of length \( a \) metres and \( b \) metres are moving in opposite directions at \( u \) m/s and \( v \) m/s, then time taken by the trains to cross each other = \((a + b)/(u+v)\) sec.

8. If two trains of length \( a \) metres and \( b \) metres are moving in the same direction at \( u \) m/s and \( v \) m/s, then the time taken by the faster train to cross the slower train = \((a+b)/(u-v)\) sec.

9. If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take \( a \) and \( b \) sec in reaching B and A respectively, then
   \( (A's\ speeet) : (B's\ speed) = (b^{1/2} : a^{1/2}).\)

SOLVED EXAMPLES
Ex. I. A train 100 m long is running at the speed of 30 km / hr. Find the time taken by it to pass a man standing near the railway line. (S.S.C. 2001)

Sol. Speed of the train = (30 x 5/18) m / sec = (25/3) m/sec.

Distance moved in passing the standing man = 100 m.

Required time taken = \( \frac{100}{\frac{25}{3}} \) sec = \( (100 \times \frac{3}{25}) \) sec = 12 sec

Ex. 2. A train is moving at a speed of 132 km/hr. If the length of the train is 110 metres, how long will it take to cross a railway platform 165 metres long? (Section Officers', 2003)

Sol. Speed of train = 132 x (5/18) m/sec = 110/3 m/sec.

Distance covered in passing the platform = (110 + 165) m = 275 m.

Time taken = \( \frac{275 \times \frac{3}{110}} \) sec = \( 15/2 \) sec = 7 1/2 sec

Ex. 3. A man is standing on a railway bridge which is 180 m long. He finds that a train crosses the bridge in 20 seconds but himself in 8 seconds. Find the length of the train and its speed?

Sol. Let the length of the train be \( x \) metres,

Then, the train covers \( x \) metres in 8 seconds and \( (x + 180) \) metres in 20 sec

\[ \frac{x}{8} = \frac{x + 180}{20} \leftrightarrow 20x = 8(x + 180) \leftrightarrow x = 120. \]

Length of the train = 120 m.

Speed of the train = \( \frac{120}{8} \) m/sec = \( \frac{m}{sec} \) = \( (\frac{15 x 18}{5}) \) kmph = 54 km

Ex. 4. A train 150 m long is running with a speed of 68 kmph. In what time will it pass a man who is running at 8 kmph in the same direction in which the train is going?

Sol: Speed of the train relative to man = (68 - 8) kmph
= \( (60 \times \frac{5}{18}) \text{ m/sec} = (50/3) \text{ m/sec} \)

Time taken by the train to cross the man

\( = \) Time taken by it to cover 150 m at \( 50/3 \text{ m/sec} = 150 \times \frac{3}{50} \text{ sec} = 9 \text{sec} \)

Ex. 5. A train 220 m long is running with a speed of 59 kmph. In what will it pass a man who is running at 7 kmph in the direction opposite to that in which the train is going?

Sol. Speed of the train relative to man = (59 + 7) kmph

\[ = 66 \times \frac{5}{18} \text{ m/sec} = 55/3 \text{ m/sec}. \]

Time taken by the train to cross the man = Time taken by it to cover 220 m at \( (55/3) \text{ m/sec} = (220 \times \frac{3}{55}) \text{ sec} = 12 \text{ sec} \)

Ex. 6. Two trains 137 metres and 163 metres in length are running towards each other on parallel lines, one at the rate of 42 kmph and another at 48 kmph. In what time will they be clear of each other from the moment they meet?

Sol. Relative speed of the trains = (42 + 48) kmph = 90 kmph

\[ = (90 \times \frac{5}{18}) \text{ m/sec} = 25 \text{ m/sec}. \]

Time taken by the trains to pass each other

\[ = \text{Time taken to cover} (137 + 163) \text{ m at 25 m/sec} = (300/25) \text{ sec} = 12 \text{ sec} \]

Ex. 7. Two trains 100 metres and 120 metres long are running in the same direction with speeds of 72 km/hr, In how much time will the first train cross the second?

Sol: Relative speed of the trains = (72 - 54) km/hr = 18 km/hr

\[ = (18 \times \frac{5}{18}) \text{ m/sec} = 5 \text{ m/sec}. \]

Time taken by the trains to cross each other

\[ = \text{Time taken to cover} (100 + 120) \text{ m at 5 m/sec} = (220/5) \text{ sec} = 44 \text{ sec}. \]

Ex. 8. A train 100 metres long takes 6 seconds to cross a man walking at 5
kmph in the direction opposite to that of the train. Find the speed of the train.

Sol: Let the speed of the train be x kmph.

Speed of the train relative to man = (x + 5) kmph = (x + 5) *5/18 m/sec.

Therefore 100/((x+5)*5/18)=6 \Rightarrow 30 (x + 5) = 1800 \Rightarrow x = 55

Speed of the train is 55 kmph.

Ex9. A train running at 54 kmph takes 20 seconds to pass a platform. Next it takes 12 sec to pass a man walking at 6 kmph in the same direction in which the train is going. Find the length of the train and the length of the platform.

Sol: Let the length of train be x metres and length of platform be y metres.

Speed of the train relative to man = (54 - 6) kmph = 48 kmph

= 48*(5/18) m/sec = 40/3 m/sec.

In passing a man, the train covers its own length with relative speed.

Length of train = (Relative speed * Time) = (40/3)*12 m = 160 m.

Also, speed of the train = 54 *(5/18)m / sec = 15 m / sec.

(x+y)/15 = 20 \Rightarrow x + y = 300 \Rightarrow Y = (300 - 160) m = 140 m.

Ex10. A man sitting in a train which is traveling at 50 kmph observes that a goods train, traveling in opposite direction, takes 9 seconds to pass him. If the goods train is 280 m long, find its speed.

Sol: Relative speed = 280/9 m / sec = ((280/9)*(18/5)) kmph = 112 kmph.

Speed of goods train = (112 - 50) kmph = 62 kmph.
19. BOATS AND STREAMS

IMPORTANT FACTS AND FORMULA

1. In water, the direction along the stream is called downstream and, the direction against the stream is called upstream.

2. If the speed of a boat in still water is \( u \) km/hr and the speed of the stream is \( v \) km/hr, then:
   - speed downstream = \( (u+v) \) km/hr.
   - speed upstream = \( (u-v) \) km/hr.

3. If the speed downstream is \( a \) km/hr and the speed upstream is \( b \) km/hr, then:
   - speed in still water = \( \frac{1}{2}(a+b) \) km/hr.
   - rate of stream = \( \frac{1}{2}(a-b) \) km/hr.

SOLVED EXAMPLES

EX.1. A man can row upstream at 7 kmph and downstream at 10 kmph. Find man’s rate in still water and the rate of current.

Sol. Rate in still water = \( \frac{1}{2}(10+7) \) km/hr = 8.5 km/hr.
Rate of current = \( \frac{1}{2}(10-7) \) km/hr = 1.5 km/hr.

EX.2. A man takes 3 hours 45 minutes to row a boat 15 km downstream of a river and 2 hours 30 minutes to cover a distance of 5 km upstream. Find the speed of the river current in km/hr.

Sol. rate downstream = \( \frac{15}{3 \frac{3}{4}} \) km/hr = \( \frac{15 \times 4}{15} \) km/hr = 4 km/hr.
Rate upstream = \( \frac{5}{2 \frac{1}{2}} \) km/hr = \( \frac{5 \times 2}{5} \) km/hr = 2 km/hr.
Speed of current = \( \frac{1}{2}(4-2) \) km/hr = 1 km/hr.

EX.3. A man can row 18 kmph in still water. It takes him thrice as long to row up as to row down the river. Find the rate of stream.

Sol. Let man’s rate upstream be \( x \) kmph, then, his rate downstream = 3 \( x \) kmph.
\( 2x = 18 \) or \( x = 9 \).
Rate upstream = 9 km/hr, rate downstream = 27 km/hr.
Hence, rate of stream = \( \frac{1}{2}(27-9) \) km/hr = 9 km/hr.

EX.4. There is a road beside a river. Two friends started from a place A, moved to a temple situated at another place B and then returned to A again. One of them moves on a cycle at a speed of 12 km/hr, while the other sails on a boat at a speed of 10 km/hr...
km/hr. If the river flows at the speed of 4 km/hr, which of the two friends will return to place A first?
Sol. Clearly the cyclist moves both ways at a speed of 12 km/hr.
The boat sailor moves downstream @ (10+4)i.e., 14 km/hr and upstream @ (10-4)i.e., 6 km/hr.
So, average speed of the boat sailor = \[ \frac{2 \times 14 \times 6}{14 + 6} \text{ km/hr} \]
= \[ \frac{42}{5} \text{ km/hr} = 8.4 \text{ km/hr} \].
since the average speed of the cyclist is greater, he will return to A first.

EX.5. A man can row 7 ½ kmph in still water. If in a river running at 1.5 km/hr an hour, it takes him 50 minutes to row to a place and back, how far off is the place?
Sol. Speed downstream = (7.5 + 1.5) km/hr = 9 km/hr;
Speed upstream = (7.5 - 1.5) kmph = 6 kmph.
Let the required distance be x km. then,
\[ \frac{x}{9} + \frac{x}{6} = \frac{50}{60} \]
2x + 3x = (5/6 * 18)
5x = 15
x = 3.
Hence, the required distance is 3 km.

EX.6. In a stream running at 2 kmph, a motor boat goes 6 km upstream and back again to the starting point in 33 minutes. Find the speed of the motor boat in still water.
Sol. Let the speed of the motor boat in still water be x kmph. then,
\[ \frac{6}{x+2} + \frac{6}{x-2} = \frac{33}{60} \]
11x^2 - 420x - 44 = 0
11x^2 - 242x + 2x - 44 = 0
(x - 22)(11x + 2) = 0
x = 22.

EX.7. A man can row 40 km upstream and 55 km downstream in 13 hours also, he can row 30 km upstream and 44 km downstream in 10 hours. Find the speed of the man in still water and the speed of the current.
Sol. Let rate upstream = x km/hr and rate downstream = y km/hr.
Then, \[ \frac{40}{x} + \frac{55}{y} = 13 \] ...(i) and \[ \frac{30}{x} + \frac{44}{y} = 10 \]
Multiplying (ii) by 4 and (i) by 3 and subtracting, we get: \[ \frac{11}{y} = 1 \] or \[ y = 11 \].
Substituting \( y = 11 \) in (i), we get: \( x = 5 \).
Rate in still water = \( \frac{1}{2} (11 + 5) \) kmph = 8 kmph.
Rate of current = \( \frac{1}{2} (11 - 5) \) kmph = 3 kmph.
20. ALLIGATION OR MIXTURE

IMPORTANT FACTS AND FORMULA

1. **Alligation**: It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.

2. **Mean Price**: The cost price of a unit quantity of the mixture is called the mean price.

3. **Rule of Alligation**: If two ingredients are mixed, then

\[
\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{\text{C.P. of dearer} - \text{Mean price}}{\text{Mean price} - \text{C.P. of cheaper}}
\]

We present as under:

<table>
<thead>
<tr>
<th>C.P. of a unit quantity of cheaper</th>
<th>C.P. of a unit quantity of dearer</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c)</td>
<td>(d)</td>
</tr>
</tbody>
</table>

\[\begin{align*}
(m) \\
(d-m) \quad (m-c)
\end{align*}\]

\[
\therefore \, \text{(Cheaper quantity)} : \text{(Dearer quantity)} = (d - m) : (m - c).
\]

4. Suppose a container contains x units of liquid from which y units are taken out and replaced by water. After n operations the quantity of pure liquid = \[x(1-y/x)^n\] units.

SOLVED EXAMPLES

**Ex. 1. In what ratio must rice at Rs. 9.30 per kg be mixed with rice at Rs. 10.80 per kg so that the mixture be worth Rs. 10 per kg?**

**Sol.** By the rule of alligation, we have:

- C.P. of 1 kg rice of 1st kind (in paise) = 930
- C.P. of 1 kg rice of 2nd kind (in paise) = 1080
- Mean price = 1000

\[
\text{Ratio} = \frac{1080 - 1000}{1000 - 930} = \frac{80}{70} = \frac{8}{7}
\]

\[
\therefore \text{Cheaper : Dearer} = 8 : 7.
\]
\[ \text{Mean price (in paise)} \]

\[ \begin{array}{c}
930 \\
1080 \\
80 \\
70 \\
\end{array} \]

\[ \therefore \text{Required ratio} = 80 : 70 = 8 : 7. \]

Ex. 2. How much water must be added to 60 litres of milk at 1 \( \frac{1}{2} \) litres for Rs. 2 so as to have a mixture worth Rs. 10 \( \frac{2}{3} \) a litre?

Sol. C.P. of 1 litre of milk = Rs. \((20 \times \frac{2}{3})\) = Rs. \(\frac{40}{3}\)

\[ \begin{array}{c}
\text{C.P. of 1 litre of milk} \\
0 \\
\text{C.P. of 1 litre of milk} \\
\frac{40}{3} \\
\end{array} \]

\[ \text{Mean price (Rs.} \frac{32}{3} \text{)} \]

\[ \begin{array}{c}
(\frac{40}{3} - \frac{32}{3}) = \frac{8}{3} \\
(\frac{32}{3} - 0) = \frac{32}{3} \\
\end{array} \]

\[ \therefore \text{Ratio of water and milk} = \frac{8}{3} : \frac{32}{3} = \frac{8}{3} : \frac{32}{3} = 1 : 4 \]

\[ \therefore \text{Quantity of water to be added to 60 litres of milk} = \left[ \frac{1}{4} \times 60 \right] \text{litres} = 15 \text{ litre} \]

Ex. 3. In what ratio must water be mixed with milk to gain 20% by selling the mixture at cost price?

Sol. Let C.P. of milk be Re. 1 per litre.

Then, S.P. of 1 litre of mixture = Re. 1.

Gain obtained = 20%.

\[ \therefore \text{C.P. of 1 litre of mixture} = \text{Rs.} \left[ \frac{(100/120) \times 1}{1} \right] = \text{Rs.} \frac{5}{6} \]
By the rule of alligation, we have:

\[
\begin{array}{ccc}
\text{C.P. of 1 litre of water} & \text{C.P. of 1 litre of milk} \\
0 & \text{Re. 1} \\
(\text{Re. } 5/6) & \\
(1 - (5/6)) = 1/6 & ((5/6) - 0) = 5/6 \\
\end{array}
\]

\[
\text{Ratio of water and milk} = 1/6 : 5/6 =
\]

**Ex. 4. How many kgs. of wheat costing Rs. 8 per kg must be mixed with 86 kg of rice costing Rs. 6.40 per kg so that 20% gain may be obtained by Belling the mixture at Rs. 7.20 per kg?**

**Sol.** S.P. of 1 kg mixture = Rs. 7.20, Gain = 20%.

\[
\therefore \text{C.P. of 1 kg mixture} = \text{Rs. } \left(\frac{100}{120} \times 7.20\right) = \text{Rs. } 6.
\]

By the rule of alligation, we have:

\[
\begin{array}{ccc}
\text{C.P. of 1 kg wheat of 1st kind} & \text{C.P. of 1 kg wheat of 2nd kind} \\
(800p) & (540p) \\
\text{Mean price} & \\
(600p) & \\
60 & 200 \\
\end{array}
\]


Let x kg of wheat of 1st kind be mixed with 36 kg of wheat of 2nd kind.

Then, 3 : 10 = x : 36 or 10x = 3 * 36 or x = 10.8 kg.
Ex. 5. The milk and water in two vessels $A$ and $B$ are in the ratio $4:3$ and $2:3$ respectively. In what ratio, the liquids in both the vessels be mixed to obtain a new mixture in vessel $C$ containing half milk and half water?

**Sol.** Let the C.P. of milk be Re. 1 per litre

- Milk in 1 litre mixture of $A = \frac{4}{7}$ litre;
- Milk in 1 litre mixture of $B = \frac{2}{5}$ litre;
- Milk in 1 litre mixture of $C = \frac{1}{2}$ litre

C.P. of 1 litre mixture in $A = \text{Re.} \frac{4}{7}$; C.P. of 1 litre mixture in $B = \text{Re.} \frac{2}{5}$

Mean price = Re. $\frac{1}{2}$

By the rule of alligation, we have:

\[
\begin{array}{c|c|c}
\text{C.P. of 1 litre mix. in A} & \text{C.P. of 1 litre mix. in B} \\
(\frac{4}{7}) & (\frac{2}{5}) \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\text{Mean price} & \text{C.P. of mixture in C} \\
(\frac{1}{2}) & (\frac{1}{10}) (\frac{1}{14}) \\
\end{array}
\]

Required ratio = $1/10 : 1/14 = 7 : 5$